Continuity Equation for flow in a Pipe

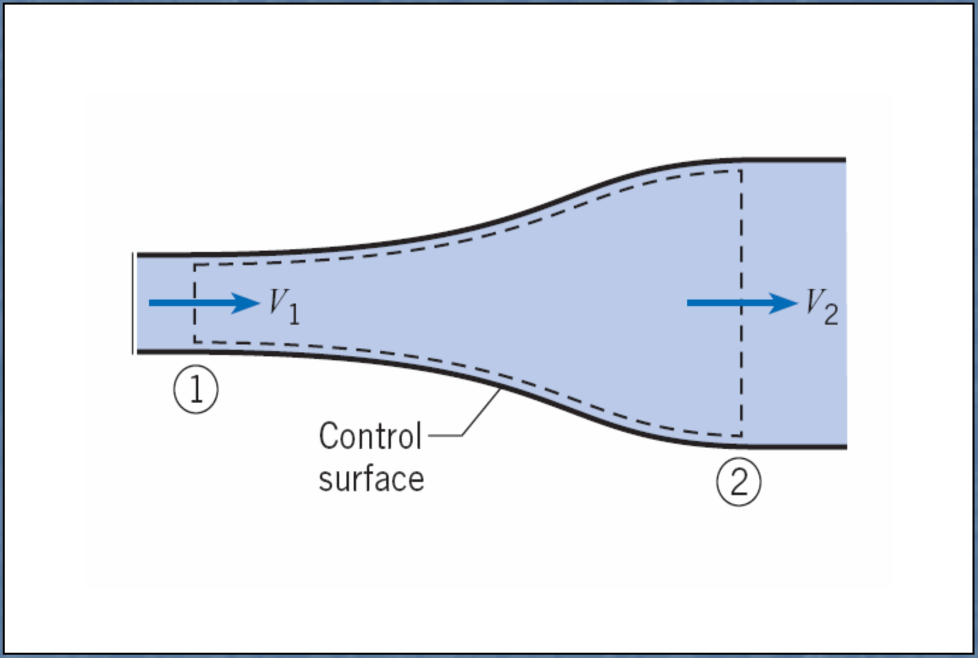
If the flow is steady, then $\frac{dM_{CV}}{dt} = 0$

i.e. $\dot{m}_1 = \dot{m}_2$

$$(\rho AV)_1 = (\rho AV)_2$$

$$(AV)_1 = (AV)_2$$

$$\dot{Q}_1 = \dot{Q}_2$$



CONTROL VOLUME APPROACH

A 120-cm pipe is in series with a 60-cm pipe. The rate of flow of water in the system of pipes is 2 m^3/s. What is the velocity of flow in each pipe?

Solution

$$Q = V_{120}A_{120}$$

Here $\quad Q = 2\ m^3/s \quad$ and $\quad A_{120} = \frac{\pi}{4} \times (1.20)^2 = 1.13\ m^2$

Therefore

$$V_{120} = \frac{Q}{A_{120}} = \frac{2\ m^3/s}{1.13\ m^2} = 1.77\ m/s \quad \triangleleft$$

Also

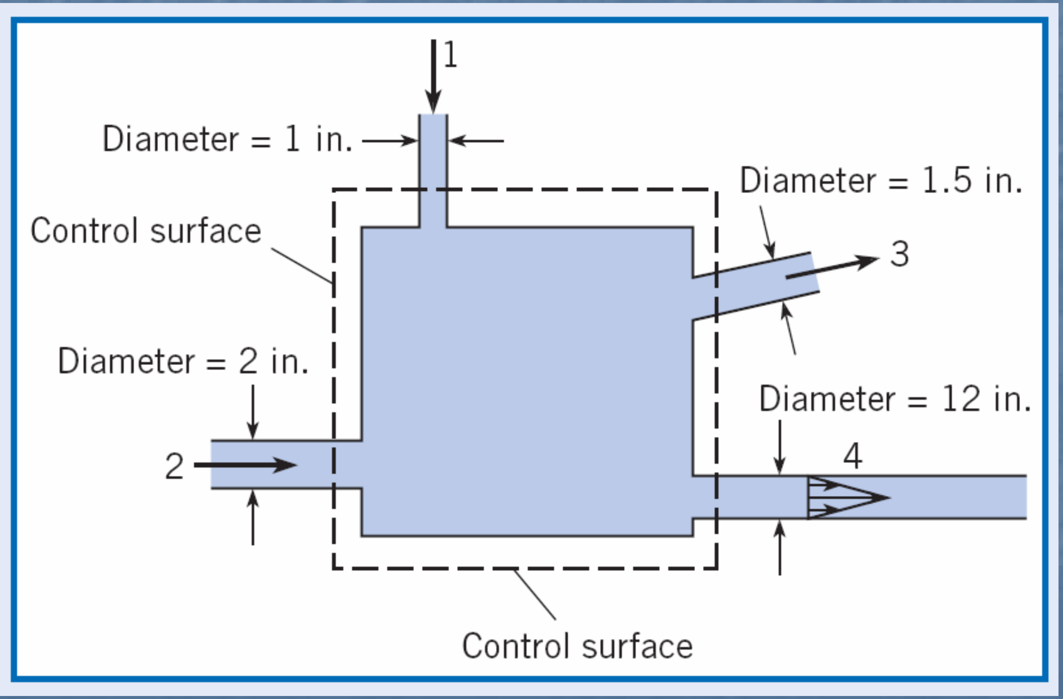
$$V_{120}A_{120} = V_{60}A_{60}$$

So

$$V_{60} = V_{120}\left(\frac{A_{120}}{A_{60}}\right) = V_{120}\left(\frac{120}{60}\right)^2 = 7.08\ m/s \quad \triangleleft$$

Example 5.10a (p. 161)

As shown in the accompanying figure, water flows steadily into a tank through pipes 1 and 2 and discharges at a steady rate out of the tank through pipes 3 and 4. The mean velocity of inflow and outflow in pipes 1, 2, and 3 is 50 ft/s, and the hypothetical outflow velocity in pipe 4 varies linearly from zero at the wall to a maximum at the center of the pipe. What are the mass rate of flow and the discharge from pipe 4, and what is the maximum velocity in pipe 4?



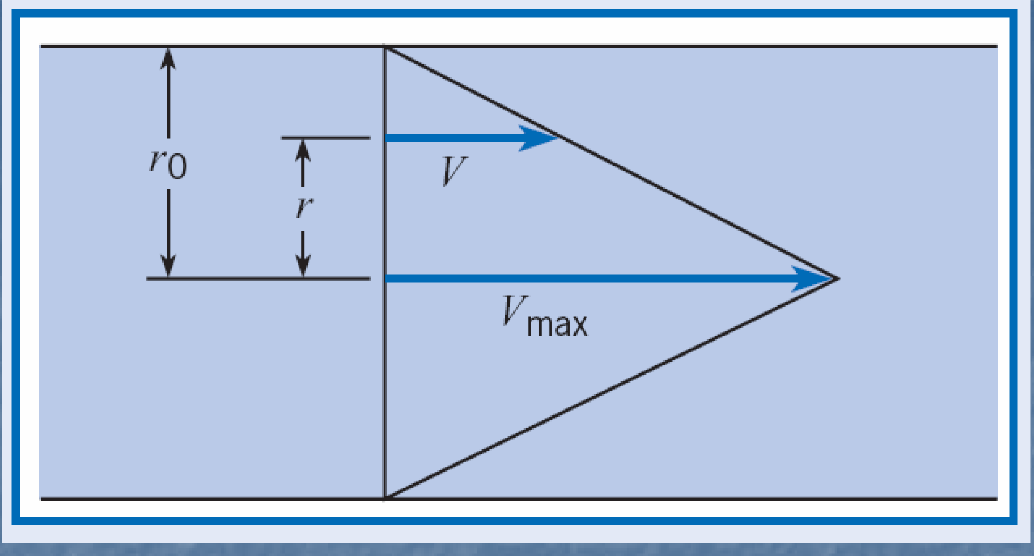
Find the mass and volume flow rate at station (4), also V(max) at (4)?

$$\sum_{cs} Q_o = \sum_{cs} Q_i$$

$$Q_3 + Q_4 = Q_1 + Q_2$$

Solving for Q_4, we have

$$\begin{aligned} Q_4 &= Q_1 + Q_2 - Q_3 \\ &= V_1 A_1 + V_2 A_2 - V_3 A_3 \\ &= 50 \times \frac{\pi}{4}(D_1^2 - D_2^2 + D_3^2) \\ &= 50 \times \frac{\pi}{4} \times \frac{1}{144}(1 + 4 - 2.25) \\ Q_4 &= 0.750 \text{ ft}^3/\text{s} \end{aligned}$$



Therefore, by proportions we have

$$\frac{V}{r_0 - r} = \frac{V_{max}}{r_0} \qquad \text{or} \qquad V = V_{max}\left(1 - \frac{r}{r_0}\right)$$

Thus

$$Q = \int_A V\,dA = \int_0^{r_0} V_{max}\left(1 - \frac{r}{r_0}\right) 2\pi r\,dr$$

$$Q = 2\pi V_{max} \int_0^{r_0} \left(1 - \frac{r}{r_0}\right) r\,dr = 2\pi V_{max} r_0^2\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \tfrac{1}{3}\pi r_0^2 V_{max}$$

Thus
$$V_{max} = \frac{Q}{\frac{1}{3}\pi r_0^2} = \frac{0.75\ \text{ft}^3/\text{s}}{\frac{1}{3}\pi \times \left(\frac{1}{2}\right)^2\ \text{ft}^2} = 2.86\ \text{ft/s} \qquad \triangleleft$$

Example 5.11 (p. 163)

Air flows at a steady rate in a pipe in which the cross-sectional area doubles between stations 1 and 2 and the velocity is reduced by a factor of 1.95. Find the percentage change in air density.

Solution Assume a one-dimensional flow. The continuity equation between the two stations is

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

The density ratio can be expressed as

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} \frac{A_1}{A_2}$$

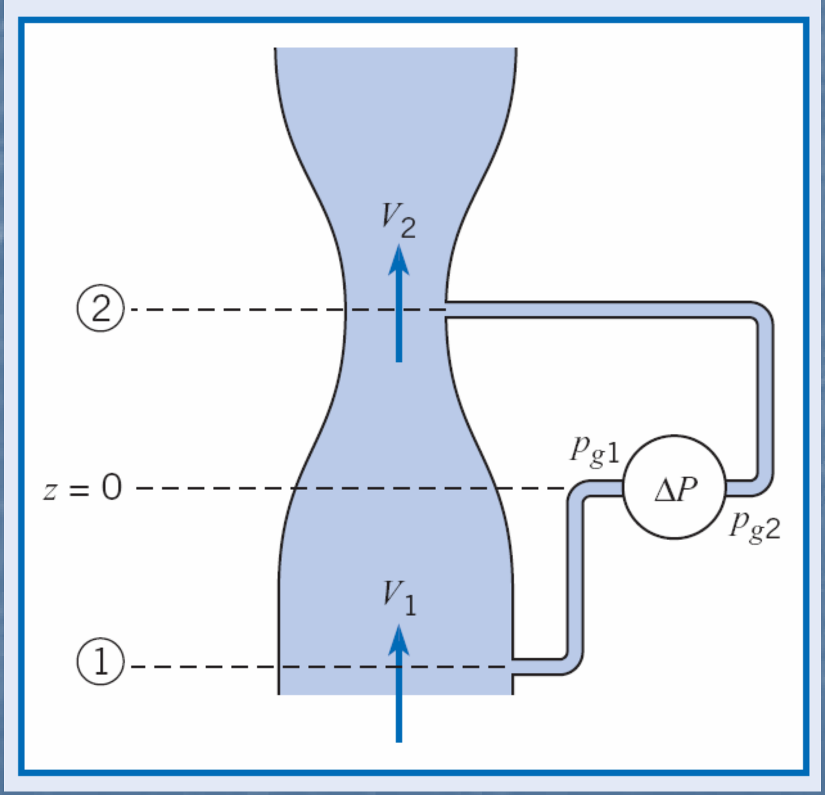
Substituting in the values,

$$\frac{\rho_2}{\rho_1} = \frac{1.95}{1} \frac{1}{2} = 0.975$$

The density is reduced by 0.025 or 2.5%.

Example 5.12 (p. 163)

Water with a density of 1000 kg/m^3 flows through a vertical venturimeter as shown. A pressure gage is connected across two taps in the pipe (1) and the throat (2). The area ratio A_{throat}/A_{pipe} is 0.5. The velocity in the pipe is 10 m/s. Find the pressure difference recorded by the pressure gage. Assume the flow has a uniform velocity distribution and that viscous effects are not important.



Find the pressure difference recorded by the pressure gauge?

Solution The Bernoulli equation is used to relate the pressures at stations (1) and (2):

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

The steady-flow continuity equation is used to find the velocity ratio between taps 2 and 1:

$$\frac{V_2}{V_1} = \frac{A_1}{A_2}$$

Define the zero elevation at the gage location. The water in the lines from the pressure taps to the gage is static, so the pressure at the upstream connection to the gage, p_{g_1}, is

$$p_{g_1} = p_1 + \gamma z_1$$

and the pressure at the downstream connection, p_{g_2}, is

$$p_{g_2} = p_2 + \gamma z_2$$

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The Bernoulli equation simplifies to

$$p_{g_1} + \rho\frac{V_1^2}{2} = p_{g_2} + \rho\frac{V_2^2}{2}$$

and the pressure across the gage is

$$p_{g_1} - p_{g_2} = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$\Delta p_g = \rho\frac{V_1^2}{2}\left(\frac{V_2^2}{V_1^2} - 1\right)$$

Using the steady-flow continuity equation,

$$\Delta p_g = \rho\frac{V_1^2}{2}\left(\frac{A_1^2}{A_2^2} - 1\right)$$

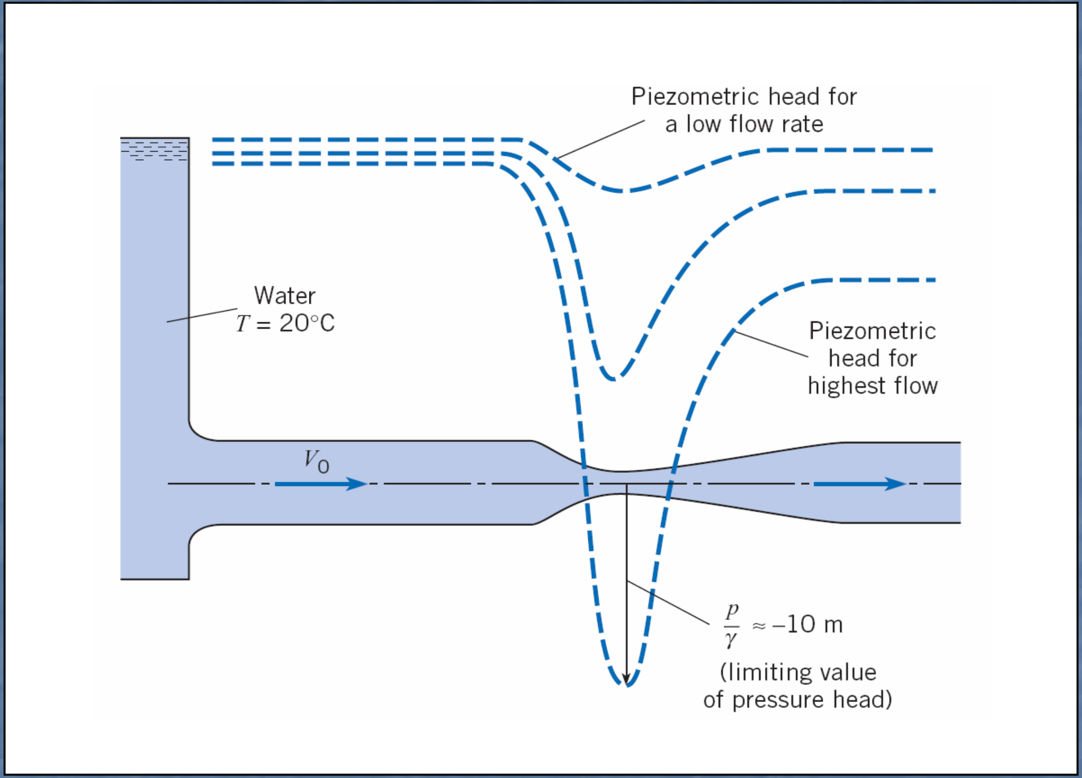
$$= \frac{1000\ \text{kg/m}^3 \times 10^2(\text{m}^2/\text{s}^2)}{2}(2^2 - 1)$$

$$= 150\ \text{kPa}$$

◁

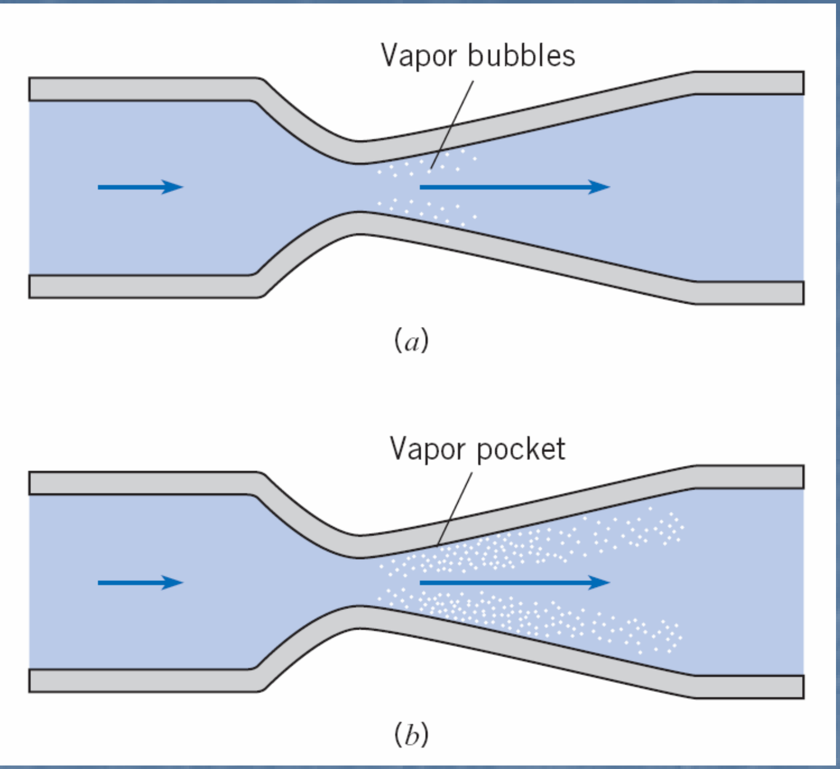
CAVITATION

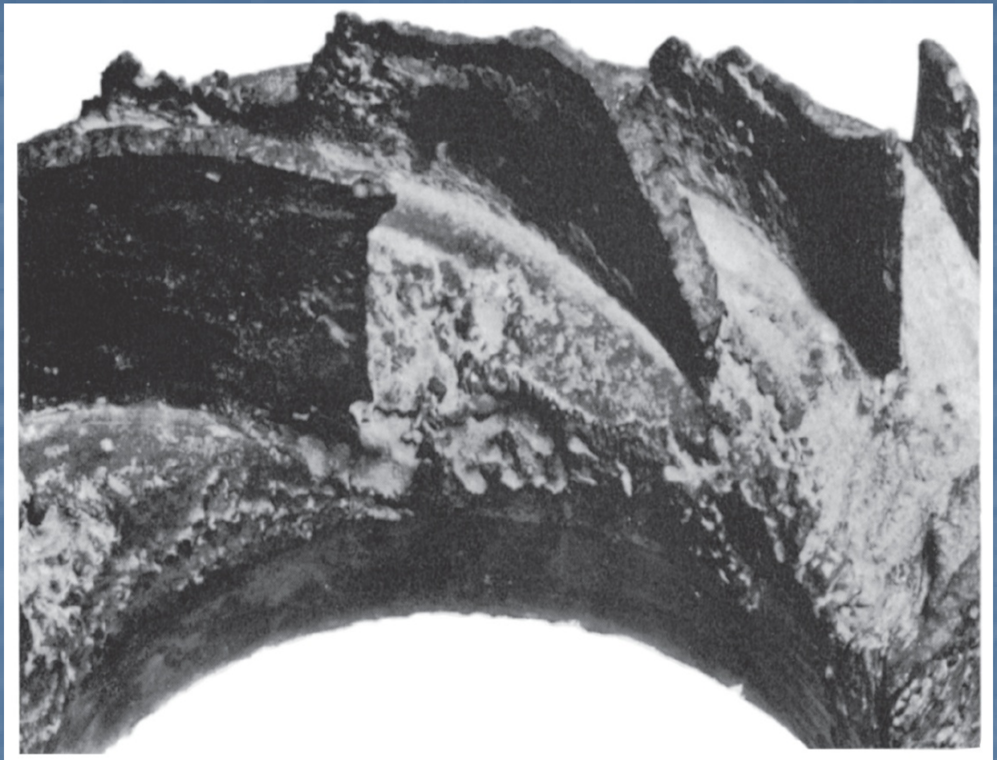
Cavitation occur when the fluid pressure is reduced to the local Vapour pressure and boiling occurs.



Flow through pipe restriction: Variation of piezometric head.

CAVITATION





Formation of vapor bubbles in the process of cavitation.
(a) Cavitation.
(b) (b) Cavitation – higher flow rate.

Cavitation damage to impeller of a centrifugal pump.

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CAVITATION INDEX:

Is the negative of coefficient of pressure = $\sigma = \dfrac{p_0 - p_V}{\frac{1}{2}\rho V_1^{\,2}}$

Cavitation is also affected by the followings:

1. Contaminant gases
2. Turbulent flow
3. Viscosity

CONTROL VOLUME APPROACH

Differential form of continuity equation

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=\frac{\partial \rho}{\partial t}$$

If the flow is steady, the above equation becomes,

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

If the flow is incompressible $\rho = Constant$ **, the above equation becomes,**

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Example 5.13 (p. 169)

The expression $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ is said to represent the velocity for a two-dimensional incompressible flow. Check it to see whether it satisfies continuity.

Solution

$$u = 10x \quad \text{so} \quad \frac{\partial u}{\partial x} = 10$$

$$v = -10y \quad \text{so} \quad \frac{\partial v}{\partial y} = -10$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10 - 10 = 0$$

Continuity is satisfied.

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END OF LECTURE (5)